A. D. Kondrat'ev

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A physical model of the dynamics of the formation and separation of bubbles during the discharge of vapor into an underheated liquid is discussed. Different regimes of formation of vapor bubbles in a liquid layer are discovered. The results of the analytic investigation are in satisfactory agreement with experimental data.

The volumetric gas (vapor) content and the area of the phase interface serve as the main characteristics of a two-phase bubbling layer. When the thickness of the layer is relatively small, they depend on the rate of growth of bubbles from the opening, their separation size, and the separation frequency. Various relations [1-6] and generalizing functions [7-10], which are in satisfactory agreement with test data, have been proposed for calculating the separation sizes of gas bubbles. The assumption that the inertial force of the liquid, due to the time variation of the radial velocity of growth of the bubble surface, is proportional to the volume of the bubble that forms was used in the derivation of a general function in [7]. In contrast to gas bubbles, the surface of a vapor bubble pulsates during condensation in an underheated liquid. The frequency of these pulsations grows while the amplitude decreases with an increase in the velocity of vapor discharge from the opening and in the amount of underheating of the liquid below the saturation temperature [11-15]. The mechanism of development of the pulsations is still insufficiently clear. They point to the radical difference between the processes of formation of vapor and gas bubbles, however. The latter is also confirmed in a comparison of the relations for determining the separation sizes of gas and vapor bubbles, containing different determining parameters and complexes [7, 16]. The available empirical recommendations [16, 17] for the calculation of vapor bubbles are restricted by the experimental conditions, not having sufficiently clear and physically justified limits of application, and they cannot pretend to extensive use in practical calculation.

In the analysis of equilibrium gas bubbles one usually assumes that at the instant of separation from an opening, a bubble is under the action of the following main forces [7]:
buoyant force

$$
F_{g}=\pi d^{3}\left(\gamma^{\prime}-\gamma^{\prime \prime}\right) / 6
$$

surface tension

$$
F_{\sigma}=\pi d_{c} \sigma
$$

hydrodynamic gas pressure

$$
F_{p}=\gamma^{\prime \prime} w_{c}^{2} \pi d_{c}^{2} / 4 g
$$

and the inertial force of the liquid

$$
F_{\mathbf{i}}=d(m u) / d \tau
$$

High-speed photography of the separation of a vapor bubble in an underheated liquid [12, $13,17]$ showed that, just as for gas bubbles, an inertial effect in the rear zone, due to the closure of the surface of the bubble after it separates from the connecting column [8], is clearly recorded (Fig. 1), so that in the analysis of the equilibrium of a condensing vapor bubble the inertial force of the liquid was retained in the system of acting forces.

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Fig. 1. Photograph of the separation of a vapor bubble in an underheated liquid; $\mathrm{d}_{\mathrm{c}}=0.003 \mathrm{~m}, \mathrm{~h}=0.09 \mathrm{~m}, \mathrm{w}_{\mathrm{c}}=$ $40 \mathrm{~m} / \mathrm{sec}, \Delta \mathrm{T}=11.5^{\circ} \mathrm{K}$.


Fig. 2. Stages in the dynamics of a condensing vapor bubble: 1) bubble boundary before the start of condensation; 2) interlayer of condensate; 3) liquid streamlines.

In contrast to gas bubbles, however, the condensation process also hinders the growth of a vapor bubble in an underheated liquid. The available experimental data allow us to propose the existence of three main stages in the dynamics of a vapor bubble. The appearance in the layer of a bubble with a size dependent on the counteraction of the forces of surface tension of the liquid and of the hydrodynamic pressure of the vapor belongs to the first stage (Fig. 2a). The formation at the interface of a thickening interlayer of condensate and a corresponding decrease in the size of the bubble occur in the second stage during heat transfer from the vapor through the bubble surface to the liquid (Fig. 2b). In the third stage, the layer of condensate reaches a thickness at which the intensity of heat transfer from the vapor through this layer to the liquid is insufficient to compensate for the hydrodynamic pressure of the vapor, and the size of the bubble increases again. In the process, the interlayer of "hot" condensate is forced back into the interior of the liquid by the forming bubble, while its place is taken by underheated liquid, and the condensation process resumes (Fig. 2c). The repetition of these stages results in the visible pattern of pulsations of the bubble surface. The irregularity of the pulsations, characteristic in the condensation of bubbles in a noncirculating confined volume of liquid $[12,13]$, is explained by the disturbance in the uniformity of the temperature fields in the liquid by intense mixing.


Fig. 3. Comparison of experimental (points) and calculated (curves) values of the separation diameter of a vapor bubble: $\mathrm{d}_{\mathrm{c}}=0.003 \mathrm{~m}$ : 1) $\left.\left.\mathrm{M}^{\prime}=0.1 \mathrm{~kg}, \mathrm{~h}=0.075 \mathrm{~m} ; 2\right) \mathrm{M}^{\prime}=0.2, \mathrm{~h}=0.045 ; 3\right) \mathrm{M}^{\prime}=0.15$, $\mathrm{h}=0.035 ; \mathrm{d}_{\mathrm{c}}=0.002:$ 4) $\mathrm{M}^{\prime}=0.35, \mathrm{~h}=0.075$; 5) $\mathrm{M}^{\prime}=0.46$, $\mathrm{h}=$ 0.09 ; 6) $\left.\mathrm{d}_{\mathrm{c}} \stackrel{c}{=} 0.006, \mathrm{M}^{\prime}=0.46, \mathrm{~h}=0.09 ; 7\right) \mathrm{d}_{\mathrm{c}}=0.0103, \mathrm{M}^{\prime}=0.46$, $h=0.09$; 8) calculation from empirical formula of [16]; 9, 10) from Eq. (9) with $\varepsilon_{k}=1.0$ and $\varepsilon_{k}=8.0 . \mathrm{d}, \mathrm{m} ; \Delta \mathrm{T},{ }^{\circ} \mathrm{K}$.

TABLE 1. Comparison of Analytic and Experimental Equations for Different Gas-Liquid Systems

| Gas-1iquid system | d from Eq. (10) |  | d from Eq. (11) |  | Av. correction factor A |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Delta T, \mathrm{~K}$ |  |  |  |  |
|  | 10 | 60 | 10 | 60 |  |
| Air-ethyl alcohol ( $\left.\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}\right)$ | 0,00269 | 0,00293 | 0,00175 | 0,00152 | 1,7 |
| Air-benzene $\left(\mathrm{C}_{6} \mathrm{H}_{6}\right)$ | 0,00287 | 0,00310 | 0,00136 | 0,00126 | 2,3 |
| Air-carbon tetrachloride <br> (CCl 4 ) | 0,00240 | 0,00262 | 0,00083 | 0,00071 | 3,2 |
| Air-water $\left(\mathrm{H}_{2} \mathrm{O}\right)$ | 0,00416 | 0,00433 | 0,00116 | 0,00120 | 3,7 |

With allowance for the retarding effect of the interlayer of condensate, the mass of liquid set into motion during the formation of a bubble is taken as proportional to the volume of the bubble,

$$
\begin{equation*}
m=\frac{4 \pi \varepsilon_{m}}{3 \varepsilon_{k}} R^{3} \gamma^{\prime} \tag{1}
\end{equation*}
$$

The radial velocity of decrease in the surface of the condensing vapor bubble is determined from the relation [17]

$$
\begin{equation*}
u \equiv \frac{d R}{d \tau}=2\left(\frac{3}{\pi}\right)^{0,5} \frac{\gamma^{\prime} c_{p}^{\prime} \Delta T}{\gamma^{\prime \prime} r}\left(\frac{a^{\prime} u^{\prime \prime}}{R}\right)^{0,5} \tag{2}
\end{equation*}
$$

With allowance for (1) and (2), the expression for the inertial force of the liquid during condensation of the bubble has the form

$$
\begin{equation*}
F_{\mathbf{i}}=4 \frac{\varepsilon_{m}}{\varepsilon_{k}} \frac{d}{g} \frac{\gamma^{\prime 3} c_{p}^{\prime 3} \Delta T^{2}}{\gamma^{\prime \prime 2} r^{2}} a^{\prime} u^{\prime \prime} \tag{3}
\end{equation*}
$$

We write the equilibrium equation for the instant of separation of the bubble with $\gamma^{\prime \prime} \ll \gamma^{\prime}$,

$$
\begin{equation*}
\frac{\pi d^{3}}{6} \gamma^{\prime}+\gamma^{\prime \prime} w_{c}^{2} \frac{\pi d_{c}^{2}}{4 g}=\pi d_{c} \sigma+4 \frac{\varepsilon_{r_{i}}}{\varepsilon_{k}} \frac{d}{g} \frac{\gamma^{\prime 3} c_{p}^{\prime 2} \Delta T^{\prime 2}}{\gamma^{\prime \prime 2} r^{2}} a^{\prime} \cdot i^{\prime \prime} \tag{4}
\end{equation*}
$$

or, after certain transformations,

$$
\begin{equation*}
\pi l^{3}+\pi \frac{3}{2} \frac{\gamma^{\prime \prime}}{\gamma^{\prime}} \mathrm{Fr}-6 \pi \mathrm{We}=24 \frac{\varepsilon_{m}}{\varepsilon_{k}} l^{3}\left(\frac{\gamma^{\prime}}{\gamma^{\prime \prime}}\right)^{2} \mathrm{~K}^{-2} \frac{a^{\prime} u^{\prime \prime}}{g d^{2}} . \tag{5}
\end{equation*}
$$

From Eq. (5) it follows that it is possible for different regimes to exist in the formation of vapor bubbles in an underheated liquid. For the region of relatively low Froude numbers, the separation size of a bubble is determined by the Weber number and by a complex characterizing the intensity of vapor condensation (static regime). In the other limiting case, when the Froude number is high enough, a dynamic regime sets in when the separation size of the bubble depends on the Froude number and a complex of the condensation intensity. Between these limiting regimes there is a transitional regime (with the separation and motion of bubbles in the layer), when the separation is controlled by the entire system of forces acting on the bubble, while its separation size is determined by the values of the Weber and Froude numbers and the complex of condensation intensity. Neglecting the buoyant force in the transitional regime, the relative separation diameter of a bubble was determined in accordance with (5) from the equation

$$
\begin{equation*}
\frac{d}{d_{c}}=\left\{\frac{\varepsilon_{k}}{24 \varepsilon_{m}}\left(\frac{\gamma^{\prime \prime}}{\gamma^{\prime}}\right)^{2} \mathrm{~K}^{2}\left(\frac{g d^{2}}{a^{\prime} u^{\prime \prime}}\right)\left[\pi\left(\frac{3}{2} \frac{\gamma^{\prime \prime}}{\gamma^{\prime}} \mathrm{Fr}-6 \mathrm{We}\right)\right]\right\}^{1 / 3} \tag{6}
\end{equation*}
$$

In tests it was found that the onset of the transitional regime for vapor bubbles corresponds to the jet regime of gas dispersion into a liquid, a condition of which is the rising of bubbles in the layer in a chain. In this regime the separation diameter of gas bubbles is determined by the relation [2, 3]

$$
\begin{equation*}
d=1.5 d_{c}\left(\frac{w_{c}}{u^{n}}\right)^{0,5} \tag{7}
\end{equation*}
$$

In [7] it was found that for air and nitrogen bubbles in water and methanol, the coefficient $\varepsilon_{\mathrm{m}}$ equals 32. Tests [17] showed that the rise velocity $u^{\prime \prime}$ of vapor bubbles does not differ significantly from the velocity determined from the function for gas bubbles obtained in [18],

$$
\begin{equation*}
u^{\prime \prime}=\left(\frac{\sigma^{2} g^{2}}{3 \pi \mu^{\prime} \gamma^{\prime}}\right)^{0,2} \tag{8}
\end{equation*}
$$

Because of the absence of enough reliable data on the rise velocity of vapor bubbles, it was determined from the function (8).

With allowance for (7) and the experimental value of $\varepsilon_{m}$, we represent $E q$ 。(6) in the form

$$
\begin{equation*}
\frac{d}{d_{c}}=\left\{1.95 \cdot 10^{-3} \varepsilon_{k}\left(\frac{\gamma^{\prime \prime}}{\gamma^{\prime}}\right)^{2} \mathrm{~K}^{2}\left(\frac{g w_{c} d_{c}^{2}}{a^{\prime} u^{\prime 2}}\right)\left[\pi\left(\frac{3}{2} \frac{\gamma^{\prime \prime}}{\gamma^{\prime}} \mathrm{Fr}-6 \mathrm{We}\right)\right]\right\}^{1 / 3} \tag{9}
\end{equation*}
$$

Equation (9) contains the proportionality factor $\varepsilon_{k}$, which is also needed for determining the region of existence of each regime of condensation. We do not know the values of the coefficient $\varepsilon_{k}$ or the methods of determining it. To estimate it, we used our own and the literature test data on the separation sizes of vapor bubbles in noncirculating underheated water [16, 19]. In Fig. 3 we present values of the separation diameter of a vapor bubble as a function of the degree of underheating of the water, obtained for $d_{c}=0.002-0.01 \mathrm{~m}, \mathrm{w}_{\mathrm{C}}=$ $30-50 \mathrm{~m} / \mathrm{sec}$, and $\gamma^{\prime} / \gamma^{\prime \prime}=1430$ in the tests of [19]. As can be seen from the figure, the test data are satisfactorily generalized both by an empirical formula obtained in [16] for $d_{c}=$ 0.003 m and by Eq. (9) with the values of $\varepsilon_{k}=8.0, \mathrm{~d}_{\mathrm{c}}=0.003 \mathrm{~m}$, and $\mathrm{w}_{\mathrm{c}}=40 \mathrm{~m} / \mathrm{sec}$.

The coefficient obtained remains valid in the static and transitional regimes of condensation. In the dynamic regime ( $\Delta \mathrm{T}<10^{\circ} \mathrm{K}$ ), Eq. (9) with $\varepsilon_{k}=8.0$ yields results $10-15 \%$ higher than curve 8. The latter is explained by the decrease in the intensity of vapor condensation and the decrease in the retarding effect of the interlayer of condensate in this regime. It is better to take the coefficient $\varepsilon_{k}$ as 7.0 in this regime. Curve 9 in Fig. 3 was obtained from Eq. (9) with $\varepsilon_{k}=1.0$. It is possible to neglect the influence of the retarding effect of the interlayer for all the regimes only when the velocity of motion of the bubbles in the layer exceeds the velocity of their condensation. We know of no experimental data for this case, however. It is also interesting to determine the applicability of Eq. (9) to various gas-liquid systems without condensation. For this purpose we transform Eq. (9): The condensation criterion is eliminated, the coefficient $\varepsilon_{k}$ is taken as 1.0 , and a correction factor $A$ is introduced:

$$
\begin{equation*}
\frac{d}{d_{c}}=\mathrm{A}\left\{1.95 \cdot 10^{-3}\left(\frac{\gamma^{\prime \prime}}{\gamma^{\prime}}\right)^{2}\left(\frac{g w_{c} d_{c}}{a^{\prime} u^{\prime 2}}\right)\left[\pi\left(\frac{3}{2} \frac{\gamma^{\prime \prime}}{\gamma^{\prime}} \mathrm{Fr}-6 \mathrm{We}\right)\right]\right\}^{1 / 3} . \tag{10}
\end{equation*}
$$

We compared Eq. (10) with the empirical formula [8]

$$
\begin{equation*}
\frac{d}{d_{c}}=\left[1.11-122\left(\frac{\gamma^{\prime \prime}}{\gamma^{\prime}-\gamma^{\prime \prime}}\right)^{0,96}\right]\left(\frac{\sigma}{\gamma^{\prime} R_{c}^{2}}\right)^{0,36} \tag{11}
\end{equation*}
$$

obtained in systems of air and water, ethyl alcohol, benzene, nitrobenzene, and carbon tetrachloride.

The calculations from (10) and (11) were made for $P=0.1 \mathrm{MPa}, \Delta \mathrm{T}=10-60^{\circ} \mathrm{K}, \mathrm{d}_{\mathrm{c}}=0.003$ $\mathrm{m}, \mathrm{w}_{\mathrm{c}}=40 \mathrm{~m} / \mathrm{sec}$, and $\gamma^{\prime \prime}=0.946 \mathrm{~kg} / \mathrm{m}^{3}$ and the values of $\gamma^{\prime}, \sigma, a^{\prime}$, and $\mu^{\prime}$ characteristic for each liquid [20]. It was found that the influence of $\Delta T$ on $d$ is negligibly small, while the error of averaging $d$ in a narrow range of $\Delta T$ for each liquid does not exceed $\pm 10 \%$ (see Table 1). Satisfactory agreement with (11) was achieved for values of the correction factor A ín (10) of 1.7-3.7 (depending on the properties of the liquids).

On the basis of the foregoing, we can conclude that Eq. (9), obtained on the basis of the proposed physical model of condensation, correctly reflects, in the main, the character of the dependence sought in the entire necessary range of the determining parameters and for different vapor (gas)-liquid systems.

## NOTATION

d, separation diameter of a bubble; $\gamma^{\prime}, \gamma^{\prime \prime}$, specific weight of the liquid and vapor (gas); $d_{c}$, diameter of the opening; $\sigma$, surface-tension coefficient of the liquid; $w_{c}$, velocity of discharge of the vapor (gas) from the opening; $g$, acceleration of gravity; m, mass of liquid, the motion of which is dependent on the radial velocity of bubble growth; $u$, radial velocity of growth of the bubble surface; $\tau$, time; $\varepsilon_{m}, \varepsilon_{k}$, proportionality factors; $R$, separation radius of a bubble; c; specific heat of the liquid; $\Delta T$, underheating of the liquid below the saturation temperature; $r$, latent heat of vaporization of the liquid; $a^{\prime}$, coefficient of thermal diffusivity of the liquid; $Z=\mathrm{d} / \mathrm{d} \mathrm{c} ; \mathrm{Fr}=\mathrm{w}_{\mathrm{c}}^{2} / \mathrm{gd}$, Froude number; We $=$ $\sigma / \gamma^{\prime} d_{c}^{2}$, Weber number; $K=r / C_{p} \Delta T$, condensationcriterion; ( $\left.\gamma^{\prime} / \gamma^{\prime \prime}\right)^{2} K^{-2}\left(a^{\prime} u^{\prime \prime} / g^{2}\right)$, complex characterizing the intensity of condensation of the vapor; $\mu$, visoosity coefficient of the liquid; $R_{C}$, radius of the opening; $P$, working pressure; $h$, thickness of the liquid layer
above the opening; $M^{\prime}$, mass of liquid in which the vapor condenses; $d_{0}, d_{i}$, initial diameter of a bubble and diameter after completion of the stage of its condensation.

## LITERATURE CITED

1. V. N. Stabnikov, "Motion of gas jets in a viscous liquid," Khim. Mashinostr., I, No. 1, 6-15 (1938).
2. L. S. Aksel'rod and V. V. Dil'man, "Bubbling at low gas velocities," Zh. Prikl. Khim. (Moscow), 27, No. 5, 485-492 (1954).
3. Ya. Tsiborovskii, Principles of Processes of Chemical Technology [in Russian], Khimiya, Leningrad (1967).
4. I. G. Malenkov, "Motion of large gas bubbles rising in a liquid," Zh. Prikl. Mekh. Tekh. Fiz., No. 6, 130-134 (1968).
5. H. D. Mendelson, "The prediction of bubble terminal velocities from wave theory," AIChE J., 13, No. 2, 250-253 (1967).
6. A. A. Voloshko, A. V. Vurgaft, and V. N. Frolov, "Onset of the jet regime of gas dispersion into a liquid," Inzh.-Fiz. Zh., 40, No. 2, 244-248 (1981).
7. A. A. Voloshko, A. V. Vurgaft, and V. N. Frolov, "Regimes of formation of gas bubbles in a liquid layer," Inzh.-Fiz. Zh., 35, No. 6, 1065-1071 (1978).
8. S. S. Kutateladze and M. A. Styrikovich, Hydrodynamics of Gas-Liquid Systems [inRussian], Énergiya, Moscow (1976).
9. K. Ruff, T. Pilhofer, and A. Mersmann, "Die Grenzenkriterien der Strahlbildung," Chem.-Ing.-Tech., 48, No. 9, 7.59-764 (1976).
10. G. J. Payne and R. G. H. Prince, "The transition from jetting to bubbling at a submerged orifice," Trans. Inst. Chem. Eng., 53, No. 4, 209-223 (1975).
11. A. P. Solodov and M. V. Spasskov, "Experimental investigation of the condensation of a vapor jet in a liquid stream by the method of local probing with a laser beam, " Tr . Mosk. Energ. Inst., No. 34, 85-95 (1980).
12. A. A. Voloshko, A. V. Vurgaft, and L. S. Aksel'rod, "Condensation of vapor bubbles in a liquid," Teor. Osn. Khim. Tekhnol., 7 , No. 2, 269-272 (1973).
13. M. E. Simpson and C. K. Chan, "Hydrodynamics of a subsonic vapor jet in subcooled liquid," Trans. ASME, Ser. C, No. 2, 48-55 (1982).
14. A. Jzuo, N. Hideki, and K. Michiyki, "'Pressure and fluid oscillation in vent system with steam condensation," J. Nucl. Sci., 17, No. 7, 326-332 (1980).
15. J. H. Pitts, "Steam chocking in a boiling water reactor pressure-suppression containment," Int. J. Multiphase Flow, $\underline{6}$, No. 4, 380-384 (1980).
16. A. A. Voloshko and A. V. Vurgaft, "Condensation of single vapor bubbles in a layer of underheated liquid," Inzh.-Fiz. Zh., 19, No. 2, 206-210 (1970).
17. A. D. Kondrat'ev, "Heat exchange during vapor condensation in a liquid volume," Izv. Akad. Nauk SSSR, Energet. Transport, No. 2, 154-159 (1984).
18. V. V. Kafarov, Principles of Mass Transfer [in Russian], Vysshaya Shkola, Moscow (1979).
19. A. D. Kondrat'ev, "Vapor condensation in an underheated liquid," Izv. Vyssh. Uchebn. Zaved., Energ., No. 11, 92-95 (1984).
20. S. S. Kutateladze and V. M. Borishanskii, Handbook on Heat Transfer [in Russian], Gos. Energet. Izd., Leningrad-Moscow (1959).

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